INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

MatheMusical Virtual Museum: Virtual Reality application to explore Math-Music World

Gilles Baroin
gilles@baroin.org
University of Toulouse, France

Abstract: Virtual Reality is a promising tool for pedagogy in teaching music, and the MatheMusical Virtual Museum takes this promise to a new level. This study aims to share and describe the project's concept and advancement, including expertise and collected experiences in MatheMusic. Mathemusic is the scientific field that studies the relations between math and music; the Society for Mathematics and Computation in Music regroups worldwide researchers in the field. The Mathemusical Virtual Museum is an interactive virtual experience that premiered at the Museum of Design in Atlanta (Moda) in June 2022 (Baroin & de Gérando, 2022). It contains a growing set of interactive models based on the research issued by our society. Mathemusicians have always produced models for understanding, analyzing, or computing music. Visualizing some on paper, theater, or computer screens is expected. Even if it is in a multidimensional space (3D-4D), the viewer ends up with a 2D picture or a movie while displaying these models on a computer screen. Planar projection limits the perception in the era of virtual reality nowadays. This study proposes tools and solutions to apprehend these models better and improve the viewer's immersive experience. Since the beginning of the project, user experiences were collected. Besides desperately unfit users for VR, we noticed that kids are highly engaged and immersed in the learning environment; music learners are not just improving their knowledge but also feeling inspired and hopeful about the future of music education; mathematicians and researchers are feeling like they are finally entering their creation and contributing to improving the project, thereby making significant contributions to their respective fields.

Keywords: Virtual Reality, Mathemusic, Visualization, Pedagogy, Learning Strategy

INTRODUCTION

Originating from different mathemusicians, the models help to unveil mathematical and mainly geometrical concepts behind musical structures used in existing music. Even if the composers intentionally designed their musical progressions, they may not have understood the underlying mathematical objects. Some concepts (hypersphere, quaternions, and graphs) were discovered after the classical period, and most modern composers do not know these concepts (Andreatta & Baroin, 2016).

The models feature notes, considered as 12 pitch classes : [*do, do#, ré,..si*], mapped to {0,1,.. 11} (Forte, 1977). In brief: reductio ad octavium ($do_0 \equiv do_1 \equiv do_n$) and enharmony: $do^{\#} \equiv ré^{b}$. Latin names are purposely used for notes and English notation for chords. The challenge of mapping musical concepts like notes, chords, and scales to mathematical objects is compelling. This involves representing musical elements using numbers or geometric shapes.

Harmony is analyzed as chords (a set of notes). I.e., C_{Maj} {*do, mi, sol*}. During a music piece, the chords are changed according to the score: “The Grid” in jazz and pop terminology. This approach analyzes harmonic progressions (chord changes) as trajectories in a geometric space. However, this approach might not capture the full complexity of harmony, which can also involve factors like tension and release. If one restricts the musical analysis to its harmonic path, model it as a trajectory in a dedicated geometric space.

The current models focus on representing musical objects like scales and chords but do not consider other critical musical parameters such as interpretation, acoustics, or timbre (*Klangfarbe*). These factors can significantly influence the perception of music.

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

The current mathematical models of music are limited in their ability to capture the full complexity of music. Music perception involves the mathematical relationships between notes and emotional responses, cultural influences, and individual preferences.

REVIEW OF RELATED LITERATURE

Research in music theory continues to investigate how geometric principles might be used. An example of this is the further exploration of geometric models. Mathematical graphs have evolved into a vital instrument that may be used to analyze voice leading, which refers to the movement of individual voices within chord systems (Hughes et al., 2021; Tymoczko, 2021). The Tonnetz, a geometric representation of tonal space, continues to be important in theoretical study. It is now being developed to incorporate representations of increasingly complicated harmonic transformations (Tymoczko, 2022; Crans, 2019).

Although harmony continues to be an essential component in music modeling, researchers increasingly include additional musical elements in their models. Artificial intelligence and machine learning are used to assess and categorize the timbral features of sounds, which ultimately results in the development of new computer models of musical perception (Pressnitzer et al., 2020). Work is still being done to model music's rhythmic and metrical parts, primarily emphasizing intricate rhythmic patterns (Toussaint, 2022; London, 2020).

The musicians, mathematicians, and computer scientists all accept the inherent limits of solely mathematical techniques and the significance of working together. They also appreciate the relevance of interdisciplinary approaches. Researchers are beginning to recognize the link between mathematical structures and cognitive processes in the perception of music (Vuust et al., 2022; Papadopoulos, 2022). According to Mannone and Compagno's (2020) research 2020, the combination of computational tools and music theory offers new dimensions for modeling intricate musical components.

Within the realm of music cognition research, the most recent developments include machine learning and artificial intelligence to analyze musical structures. The study “Deep Learning Techniques for Music Generation" (Briot et al., 2020) addresses the use of deep learning methods to determine the degree of similarity between different musical songs. The authors propose a geometric deep-learning model to learn similarity metrics and capture the underlying connections between musical features.

By addressing the limitations of current models and incorporating insights from related fields like machine learning and psychology, researchers can develop more comprehensive models that capture the richness and complexity of music.

Geometric representations of musical parts lend themselves readily to spatialization inside a virtual reality environment, a practice known as "spatializing music." The analysis of harmonic progressions as trajectories offers the creation of dynamic music that may react to a user's activities or movements while in virtual reality. The text's focus on limits suggests that virtual reality might add components such as acoustics and timbre, producing a more immersive experience.

Geometric models are now being incorporated into virtual reality (VR) technologies, enabling users to compose and operate music inside virtual surroundings. Musical virtual reality interfaces allow for the reimagining of musical instruments and notation inside a virtual reality environment, which may be used for creating and playing (Bailey et al., 2022). According to Wakefield and Garg (2021), interactive virtual reality spaces may assist students in conceptualizing challenging musical ideas.

The term "adaptive virtual reality music" refers to the ability of musical compositions to change in response to user input, movement, or other aspects of the virtual reality experience. According to Bown et al. (2019), virtual reality (VR) experiences use algorithms that generate music that is in sync with the images of the environment or with the activities of the person experiencing the experience.

Virtual reality (VR) can include a user's physiological data to change the soundscape in real time, impacting emotional reactions (Nacke et al., 2020). Virtual reality may also contain biofeedback. Enhancing Immersiveness with Acoustics and Timbre Virtual reality (VR) may give a solid platform to combine the physics and perception of sound, which is a problem for mathematical models. These models can be challenging to create.

Techniques such as binaural audio contribute to a heightened feeling of space and acoustic realism inside virtual reality (VR) encounters with music (Cheng et al., 2022). VR makes it possible to experiment with realistic instrument acoustics, which enhances the immersive musical experience (Alonso et al., 2022). For example, virtual reality may be used to model various instruments.

Focused research is needed on how virtual reality settings might be utilized to explore and evaluate theories of music cognition. Virtual reality (VR) has been proposed as a viable tool for immersive music experiences; nevertheless, there is still room for such research. This includes how virtual reality may assist us in comprehending ideas such as spatialized sound, embodied music perception, and emotional reactions to music.

METHODOLOGY

The transformation of mathematical and musical models into a virtual reality (VR) interactive environment and the production of 360-degree movies that can be seen in any headset comes after considerable time spent honing our skills in two-dimensional representations. The MatheMusical Virtual Museum, as shown in Figure 1, is a digital location with a gallery and chambers designated for each model. The material of this museum is constantly being updated. There will be a wide range of user engagements, from passively watching a movie to actively manipulating the model to learn about or improve it. Songs (animation and music) that have been preset and may be loaded interactively are available at the Museum. The user can import a new song in the form of a directory that includes a description, a sound file, and a list of chords that are precisely timed.

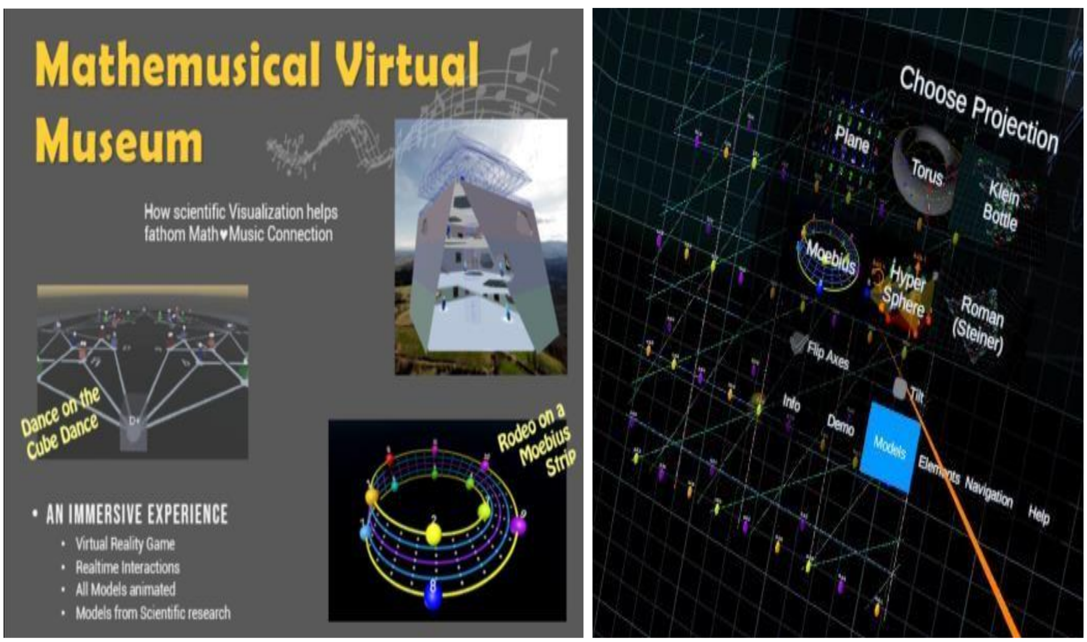

Figure 1. The MatheMusical Virtual Museum: Some possible projections

Tools

To produce 360-degree movies, a virtual camera was used within an animated 3D model, an existing flat movie was projected on a sphere around the user, or a scenario or song was recorded within the development environment. To create the Museum, the architecture, objects, and textures were designed using the usual existing tools; the scene interactions, code, and lighting are conceived inside the VR development environment. Figure 2 illustrates the flattened 360° view of the first floor.

The scenario or demo songs are Excel® generated files loaded in the application.

Meta Quest2® was used as hardware. For the software, Microsoft Excel® and Autodesk 3DSMax® were utilized to model and calculate the animations, Steinberg Cubase® for musical arrangements, and Audacity® for sound processing. After montage and postprocessing, the movie is finalized as a specific mp4 video file ready to be played in any VR environment.

Unity® was used for a game development environment and interactive applications.

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

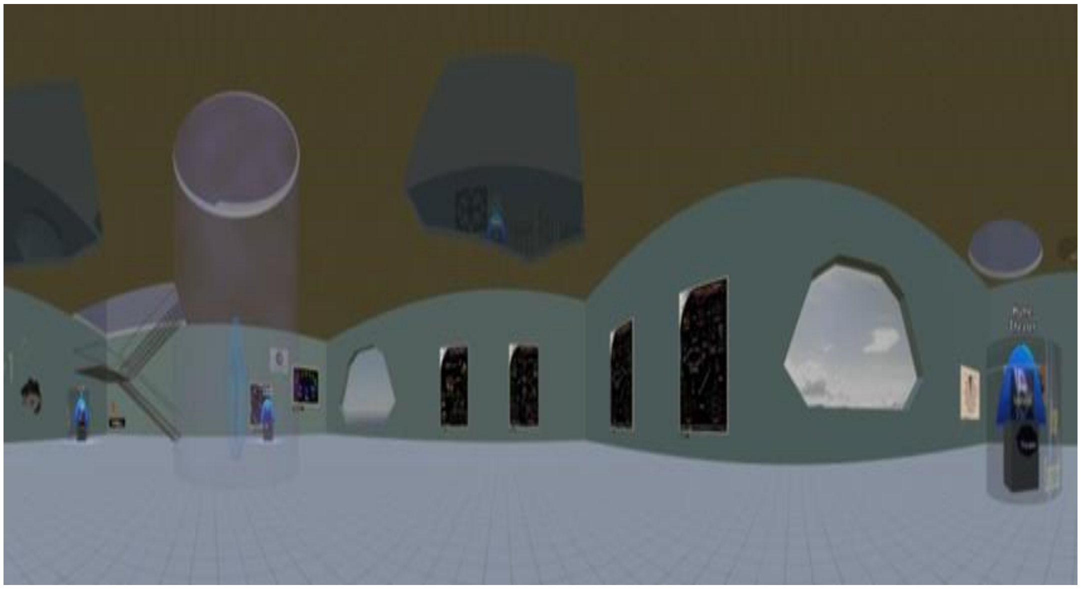

Figure 2. Flattened 360° view of the first floor

Visual design

The museum is a four-story modern exhibition hall where scenes are organized according to the complexity of their dedicated models. The architecture was designed in collaboration with children, delving into the “child’s point of view,” beta testing brought new ideas and challenges that even experienced adults would not have gotten.

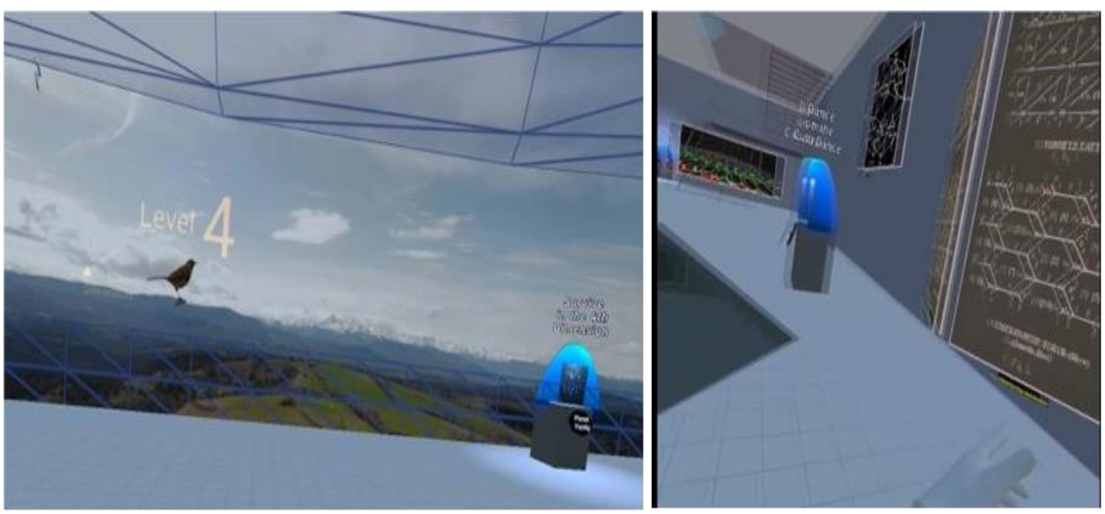

Figure 3. Terrace on the last Floor; 2nd Floor featuring the memorial to Jack Douthett

As in a VR game, the user can move and encounter different scenes featuring specific models via teleportation. Some models are interactive and customizable.

The most straightforward representations can be found on the ground floor (level 1), featuring a lift that will bring the user up to the last floor: the lair of four-dimensional models: A terrace with a 360 view of the Pyrenées

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

mountains shot with a drone; this enables the user to breathe outside versus a closed rooms ambiance. Figure 3 illustrates the memorial to Jack Douthett on the 2nd Floor while it also features the terrace on the last floor.

Original authors collaborate with us so that the shape and colors of musical elements and interaction possibilities respect their intentions.

Sound design

Scale and octave gap

Generally, each VR object representing a note is associated with a corresponding tone. On a real instrument, once the last note of the scale is reached, the next one is the first one, but one octave higher. This would not fit the mathematical models based on cyclic groups and cause octave gaps, destroying the effects of visual symmetry. Using specially designed Shepard-Risset tones (Shepard, 1964), an acoustic illusion (Risset, 1986) analog to the Escher stairs, one can play the scale up indefinitely and fathom the model's soul. Figure 4 shows the chromatic scale with an octave gap and the Shepard calculation on 4 Octaves. Shepard tones were also employed to build chords in real-time within the game engine.

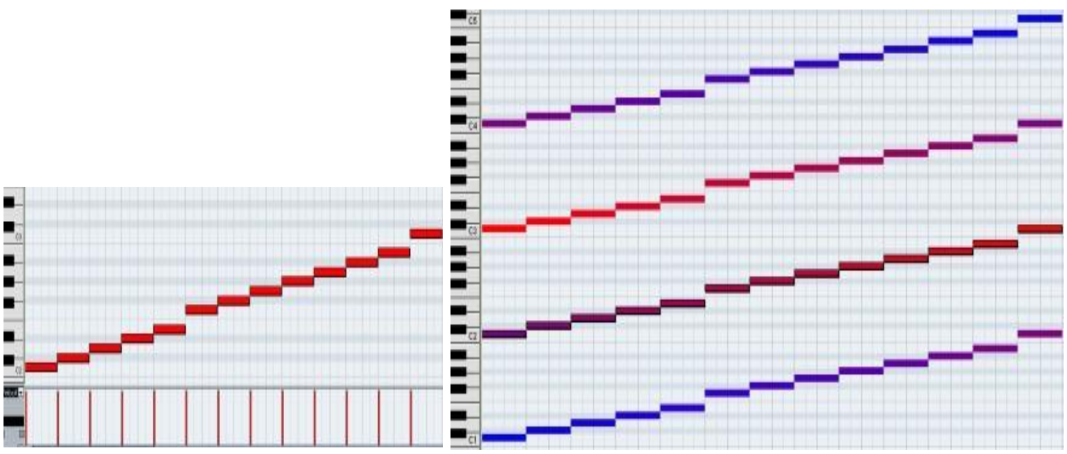

Figure 4. Chromatic scale with Octave Gap: Shepard calculation on 4 Octaves

Each note is played on 4 octaves on this scale progression with a constant sum of intensities symbolized on a blue->red scale ranging from 0->100.

Volumes were interpolated along the scale so that the two low-octave volumes varied from 0->50 to 50->100. The two upper octaves have decreasing volumes: 100->50 and 50->0. As verified, the first tone is the same as the last one, and each tone has the same intensity.

First Tone: Note#0 $0.\ do_1 + 50.\ do_2 + 100.\ do_3 + 50.\ do_4$ Last Tone: Note#12=0 $50.\ do_2 + 100.\ do_3 + 50.\ do_4 + 0.\ do_5$ All other Tones have a volume of 200, I.e. :
Triton: Note#6 (Fa#) $50.\ fa\#_1 + 50.\ fa\#_2 + 50.\ fa\#_3 + 50.\ fa\#_4$

Since each tone has the same volume, any progression of chords with the same number of notes will sound at the same intensity.

Demo songs

The songs used as demos are wave files running in the background while the animation plays.

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

Main atmosphere

A dedicated multitrack composition was produced, where each instrument plays at a different rhythm, timbre, and melody (E.g., slow deep pads, electronic effects, fast toy piano, etc.). Any combination of tracks will thus produce a unique and consonant musical arrangement.

Each track is assigned spatially to one or more dedicated locations within the building (with matching volume, range, and decay). This results in a continuously varying mood evolving with the user's position, inciting him to visit some specific places. For instance, the toy piano can be heard on all the stairs, and the deep pad is in the basement.

In addition, a selectable drum loop was played in the background (the user can choose from light bongos to a deep house drum loop). To reinforce coherence, this loop is not localized, plays in stereo, and has a tempo adapted to the central atmosphere sounds.

All calculations are performed within the game engine in real time. Extract from the main composition and the Positions of localized 3d sounds is illustrated in Figure 5.

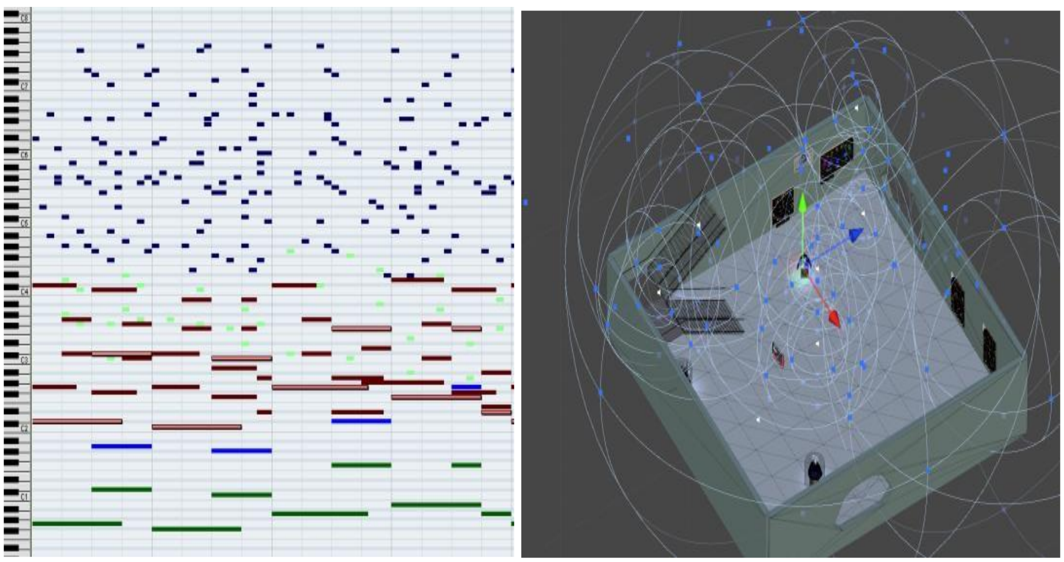

Figure 5. Extract from the main composition: Positions of localized 3d sounds

RESULTS AND DISCUSSION

The list of models is in perpetual evolution. At his time, the following performances were featured:

Level 1: Unidimensional and exceptional shows

Chromatic Circle

Newton was the most extraordinary alchemist of all time and the father of calculus, optics, and mechanics. He was the first to associate chromatism in terms of colors and tone height (note). He also associated seven planets and chemical symbols with the diatonic scale (Hofmann & Zemplén, 2020), a standard for his time.

This scene extends Newton's circle to our modern twelve-tone chromatic scale. According to human perception, each tone is associated with a color (Baroin, 2011). The circle rotates slowly, and the user can freely evolve in the scene. Figure 6 demonstrates the original version versus the revisited version of Newton's chromatic circle.

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

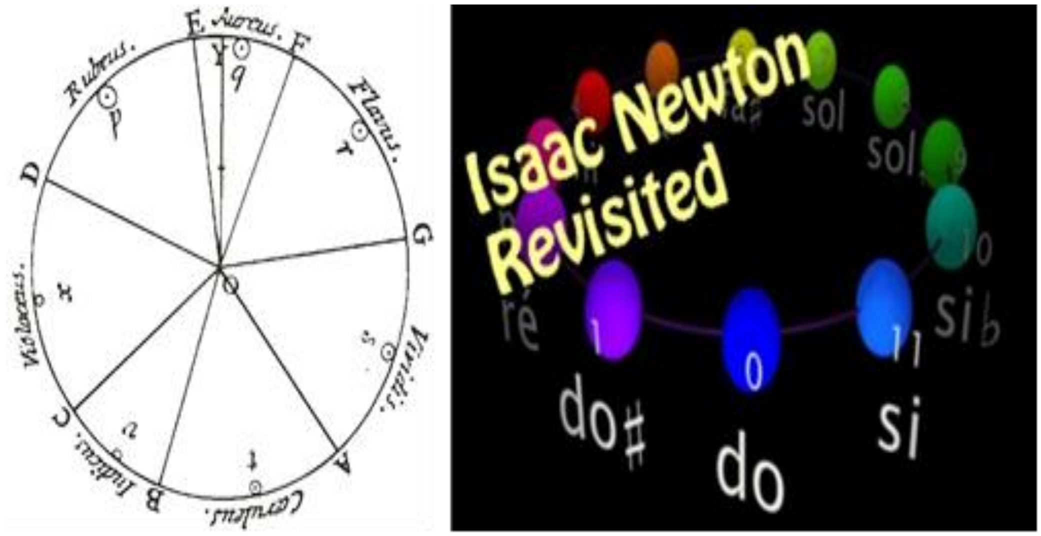

Figure 6. Original vs revisited versions of Newton's chromatic circle.

In the Pi:

In memoriam, John Sims (n.d.), math artist and the "master of π," this part will be unveiled during a special event in Florida for the next Pi Day 2023.03.14

Home theater

A scene dedicated to reading papers and watching videos is under construction.

Level 2: Graphs and 2D models

The Tonnetz

Invented by Leonard Euler and popularized during the XXth century by fellow Mathemusicians, this is the usual decomposition of the 12 tones scale into 4x3 = 3x4. This is known as the Sylow decomposition of the cyclic group $Z_{12} = Z_3 \otimes Z_4$ (Chew, 2002; Mazzola, 2002).

Therefore, each note has two coordinates associated with a two-dimensional (Shape and Color) graphical symbol. That is equivalent to labeling the graph with a Gaussian integer, $z = a + ib$, $a \in [0,1]$, $a \in [0,3]$, whose fundamental part is the position along Z_3, the imaginary one, along Z_4 (Baroin, 2011).

Musically speaking, one axis progresses by a major third (3 ½ tones), the other by a minor third (2 tones). As it shall be, but unfortunately not feasible IRL, the Tonnetz is designed on an endless plane. Its boundlessness in VR was simulated with the help of hidden teleportation and camera effects.

Therefore, the user can explore this infinite space, observe the organization, and travel according to his imagination.

The Tonnetz is made of tones, and each note symbol has an associated sound object. Since the Tonnetz of Chords (Triads = perfect Maj or Min) is the dual space of the Tonnetz of Notes (Mazzola, 2002; Chew, 2002), the user can hear multiple tones at once by adjusting the hearing range in the software. If situated, i.e., at the barycenter of [*do, mi, sol*], he will perceive the three notes simultaneously: a C_{Maj} chord. Figure 7 depicts Tonnetz by Jack Douthett (Douthett & Steinbach, 1998), and the VR immersion in endless space provides a digital abstraction.

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

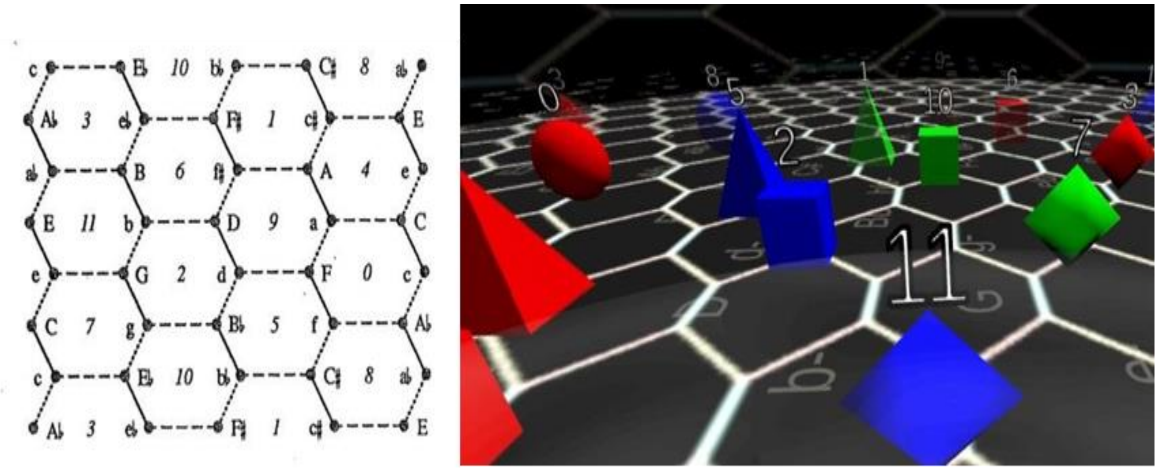

Figure 7. Tonnetz by Jack Douthett (Douthett & Steinbach, 1998) vs VR immersion in endless space

The Cube Dance

Created by Jack Douthett and Peter Steinbach, the Cube Dance (Douthett & Steinbach, 1998) features our usual 24 major and minor triads and four new chords: the augmented triads, noted '+.' An augmented triad comprises two major 3rds, i.e., C^+ [0,4,8] = [*do, mi, sol*$^\#$]. Since it is symmetrical ($C^+{}_{[0,4,8]} \equiv E^+{}_{[4,8,0]} \equiv A_b{}^+{}_{[8,0,4]}$), there are only four different augmented triads that are each connected to 3 major and three minor triads. Some user movements must be limited to respect the graph topology and experience a feasible trek. This is achieved by placing invisible walls in VR.

Since the Cube Dance is topologically embedded on a torus, other higher-dimensional projections of this graph were added in the other scenes: "The Torus of Thirds" and "Planet-4D."

Moebius strip

As in Newton's circle, this Moebius strip embeds the 12 notes on its edge. Despite the poor musical interest, it enables the user to ride on the strip. The well-known trip of an ant on a Moebius strip is only practicable in VR. Since 2D coordinates on the Moebius strip can be mapped, the model is used in other scenes (Torri of Phases) for visualizing more complex patterns. Figure 8 provides a flattened 360° view from the Moebius strip scene.

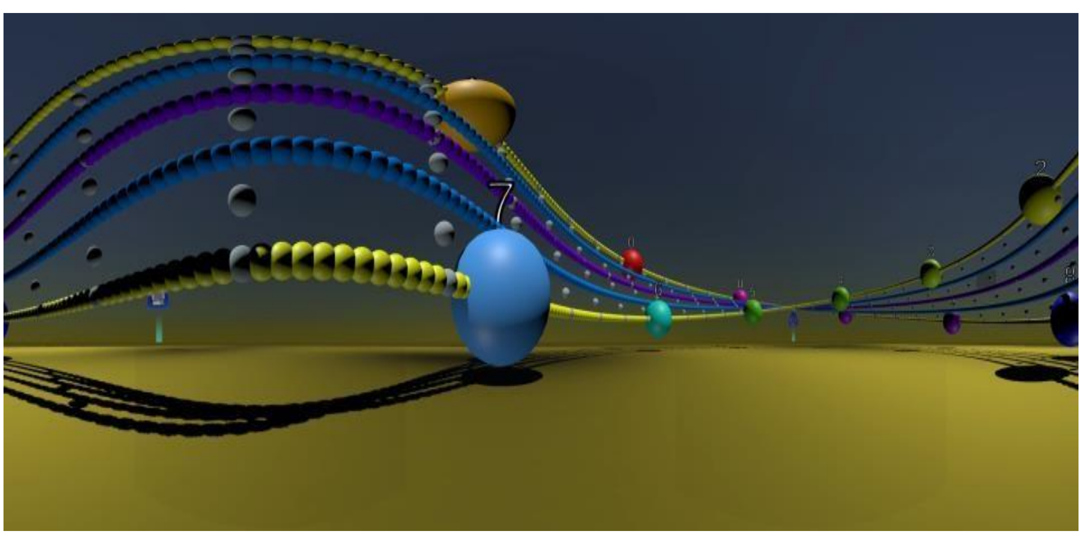

Figure 8. Flattened 360° view from Moebius strip scene

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

Level 3: 3D models

The Torus of thirds

Popularized by Guerino Mazzola (2002), this is the traditional projection of the Tonnetz on a Torus.

The user may manipulate the object with the help of the GUI. Other versions of the Torus of the third can also be observed as a 3D projection of the Torii of phases (Level 4).

Cube Harmonic

Invented by Maria Mannone (2018), it is a customized Rubik's Cube® featuring electronics and optical recognition. It is employed in the real world with an orchestra or live musicians. The system produces chords, music, and timbres; the playing tones are on the bottom face.

A Rubik's Cube had been constructed to achieve a VR counterpart, associate sounds to each facet, and let sound only the top face. Compared to the physical version, the VR one is easily customizable as the notes are stored in a file.

Figure 9. The Cube harmonic: Performance IRL vs. VR manipulation

The user can manipulate the cube via the GUI to achieve local rotation of each of the six faces or rotate the whole cube. Therefore, it has 6x2 face rotations + 3x2 global rotations.

The scene includes a game mode in which the cube is randomized and solved to create an original chord progression. Figure 9 shows the cube harmonic with the performance IRL and the VR manipulation.

Level 4: Lair of the Fourth Dimension

The mathematics behind the hyperspheres :

All hyperspheres employed are topologically equivalent to a 3-sphere; they are four-dimensional objects, and the pitches or chords were placed on their 3D surfaces (Baroin, 2011). Thus, the positions of our objects are defined by a unitary quaternion that can be interpreted, depending on the context, as:

- four spatial coordinates, $q \in \mathrm{R}^4 : q\,(x, y, z, w)$, for computing projections,
- two angles, q$(\alpha, \theta) \in \mathbb{C}\mathrm{x}\mathbb{C}: q(e^{i\alpha}, e^{i\vartheta})$, for rotations,

- a pure quaternion, $q \in \mathcal{H}$, for geometric calculations.

Knowingly used in VR and 3D to calculate rotations, quaternions as spatial Euclidian coordinates were used, then projected into 3D space for visualization (Zacharias & Velichova, 2000).

In order to achieve an additional symmetry on Euclidian distances between the 12 pitch classes, our *Spheres* are scaled differently concerning the two complex planes: they are essentially “hyper ellipsoids.” Since they are topologically equivalent to S_3, they are used to call them *Hyperspheres (*or merely *Spheres*) (Baroin, 2011). All possible positions of elementary objects (pitches or chords) are located on a particular 2D surface of the sphere.

$$F(x,y) => q;\ Q = \left(\frac{1}{\sqrt{3}}e^{i\theta}, \frac{1}{\sqrt{2}}e^{i\alpha}\right),\ (\theta,\alpha) \in \mathbb{R}^2, (mod(2\pi)).$$

Any two-dimensional coordinate were translated to a 4D one, and any 2D translation corresponds to a couple of rotations in 4D.

Since a player's evolution in 4D geometrical space is meaningless, the hypersphere was allowed to rotate in front of the user. This is similar to rotating a 3D object on a planar screen to help understand its shape. The TV screen was added in some scenes where the projection of 4D-3D-2D is performed at the usual user position, like in former flat movies (Baroin, 2020).

Planet-4D family

This is a family of hyperspace-based mathemusical models that I initiated in 2000; the reader can refer to the study of Baroin (2011), Baroin (2019), and Baroin & de Gérando (2022) for detailed mathemusical and graphical aspects.

Each pitch class is defined in a two-dimensional system that can be embedded on a torus or, in this case, a hypersphere. The two dimensions are mapped as two angles of a quaternion. The quaternionic coordinates of each tempered pitch class are:

$$Q_n = \left(\frac{1}{\sqrt{3}}e^{i\frac{2n\pi}{3}}, \frac{1}{\sqrt{2}}e^{i\frac{2n\pi}{4}}\right), n \in [0,11].$$

Due to the 4D space topology, objects must be set into motion to be understandable.

This Scene features the following models from the family:

- Planet-4D: 12 points on the surface of a Hypersphere that can represent any class of transposable musical objects.
- Hypersphere of Chords: an equivalent of the Tonnetz projected on a 4D-Sphere, dedicated to tonal music.
- Hypersphere of Chicken: including the Tonnetz of Chords’ graph.
- Hypersphere Cube Dance: embedding Jack Douthett and Peter Steinbach’s graphs.

In brief :

Planet-4D, illustrated in Figure 10, arranges the 12 tones into 4x3: $Z12 = Z3 \otimes Z4$. Each note (pitch class) features a two-dimensional symbol: shape and color. One axis is the progression by Major 3rd, the other by Minor third. Each tone has a 4D coordinate (a unitary quaternion of 2 complex numbers). Each axis corresponds to a rotation of the sphere wrt, one complex axis. Any translation on the plane (mod. 2π) is performed as a sphere rotation. The user can wander to explore this space and rotate the sphere. This scene features predefined songs: Chromatic and Fifth’s progressions for tones and chords, an extract of the famous progression in Beethoven's 9th symphony (Chew, 2002), The Hamiltonian Path in Coral *#4* by Giovanni Albini (Albini & Antonini, 2009).

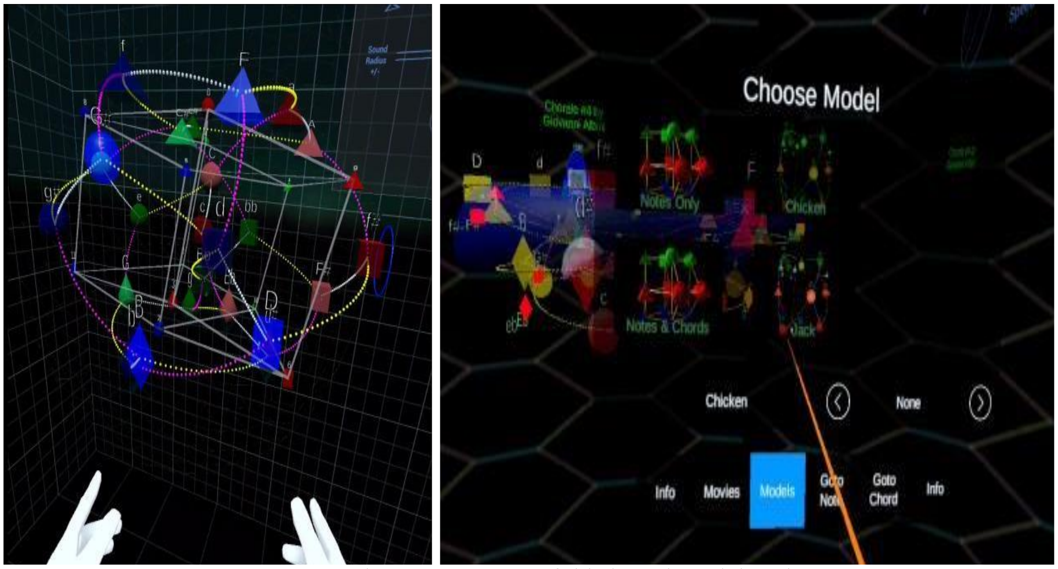

Figure 10. Planet-4D; Torus of thirds and model options

The Torii of Phases

Developed by Emmanuel Amiot (2013) and Jason Yust (2020), the Torii of Phases is a Mathemusical model using the phases φ of the (complex) Fourier coefficients of pitch classes-sets (12 notes). It provides any note-set a complex coordinate whose polar representation is $z = r.\ e^{i\varphi}$.

The amplitude parameter (r) was not considered because each set class has the same one; however, a different color for each set type was assigned. Since each set has two coordinates, it can be displayed on a plane, a torus, a hypersphere, and the rest.

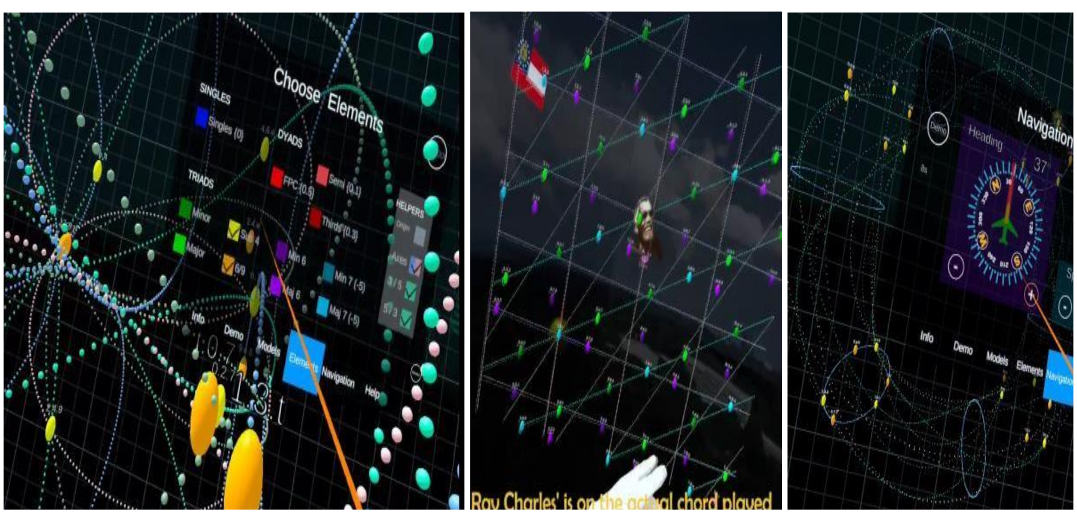

Figure 11. Torri of phases elements, Planar representation while playing Georgia on My Mind (Ray Charles), Navigation menu

The user has multiple options for :

1. Navigation: Select a direction of travel and speed
2. Selection: Select the types of chords to display (single notes, Maj, sus^4,..)

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

A harmonic analysis of "Georgia on My Mind" was presented, as illustrated in Figure 11. Ray Charles is immobile, centered on the actual chord playing, while the model moves/rotates around him. The user can switch to any representation during the demo.

Entangled Hyperspheres

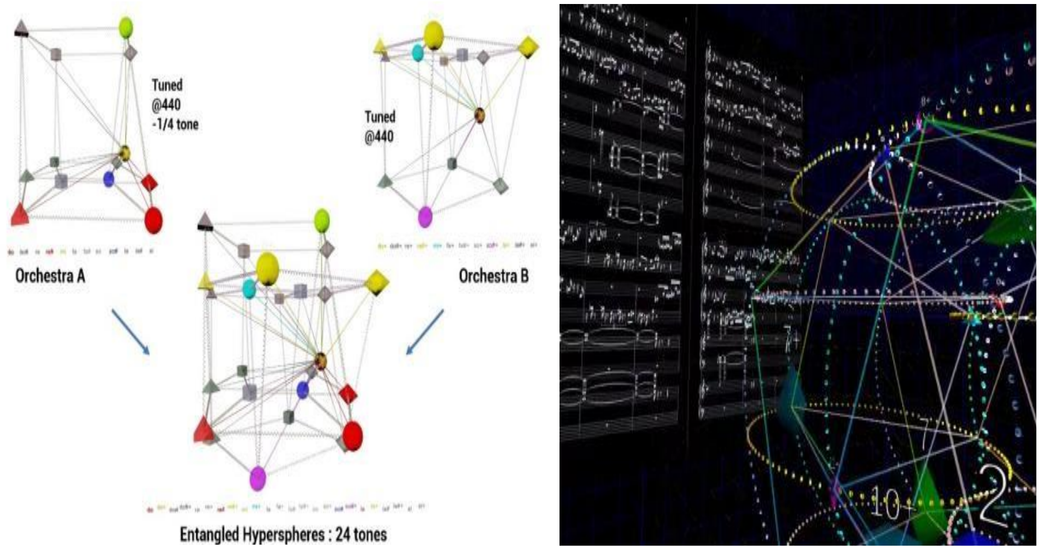

Figure 12. Entangled Hyperspheres: Construction and VR implementation with a score

As if the name was predestined, this was premiered simultaneously in Buenos Aires and Paris during the show "Virtuality" (Baroin, 2020). The microtonal performed music is composed purposefully (de Gérando. 2020).

For music in ¼th tone, the initial method, imagined by Ivan Wyschnegradsky for two pianos as described in (Jedrzejewski. 2000), can be adapted for two orchestras:

- Orchestra A is tuned at 440Hz and plays the 12 usual notes (do, do#,…)
- Orchestra B is tuned at 440Hz - ¼th tone (427.7 Hz) and plays the 12 additional notes (do+ do#+,…).

Created with S. Gérando to model 24 tones of music, two Planet-4D models are entangled: A corresponds to our scale, and B to our scale detuned by 1/4 tone.

The quaternionic coordinates of each tempered pitch class in orchestra A are:

$$Q_n = \left(\frac{1}{\sqrt{3}} e^{i\frac{2n\pi}{3}}, \frac{1}{\sqrt{2}} e^{i\frac{2n\pi}{4}}\right), n \in [0.11].$$

Since orchestra B is playing one-quarter tone above orchestra A, all symbols representing its pitch classes have to be rotated 1/24th of the whole circle along the chromatic circle, which is equivalent to adding to each coordinate a constant quaternion qc:

$$\text{qc} = \left(\frac{1}{\sqrt{3}} e^{i\frac{\pi}{12}}, \frac{1}{\sqrt{2}} e^{i\frac{2\pi}{12}}\right)$$

The coordinates of each tempered pitch class in orchestra B are therefore:

$$Q_{Bn} = Q_n + \text{qc} = \left(\frac{1}{\sqrt{3}} e^{i\frac{(8n+1)\pi}{12}+}, \frac{1}{\sqrt{2}} e^{i\frac{(6n+1)\pi}{12}}\right), n \in [0.11].$$

INTERNATIONAL JOURNAL IN INFORMATION TECHNOLOGY IN GOVERNANCE, EDUCATION AND BUSINESS
Vol. 6, No. 1, 2024
ISSN 2686-0694 (Print)
e-ISSN 2721-0030 (Online)

There were two sets of 12 quaternions sharing the exact origin; the global rotation of the system is done according to the spherical barycenter of active notes among the 24 available (Baroin & de Gérando, 2022).

As in Planet 4-D, the user freely moves and rotates the spheres; he can select the tones of A or both. The animated score is displayed while the demo song plays (de Gérando. 2020). Figure 12 depicts the Entangled Hyperspheres, which provides Construction and VR implementation with a score.

Further models in development are

- The Hypersphere of *AnySet* (Baroin, 2011), dedicated to the visualization of atonal music, enables the display of any set of tones in hyperspace and uniquely demonstrates symmetries.
- Hypersphere of Tonnetze, created with Louis Bigo, shows the generalized Tonnetze T1 to T6 on the surface of the 4D Hypersphere. It is dedicated to Tonnetz morphing.
- Hypersphere of Spectra (Baroin & de Gérando, 2012), created with Stéphane de Gérando, is an original way to display any sound according to its spectra within a 4D environment dedicated to spectral or microtonal music.
- Hypersphere of Temperaments (Baroin & Calvet, 2019), created with André Calvet, enables visualization of non-equally tempered music on a deformed Sphere (based on S3, but with varying radius).
- Geometry of Music models from Dmitry Tymoczko (Hook, 2011) - Spiral Array by Elaine Chew (Chew, 2002)

User Experiences

Since the beginning of this project, varied users have been included, who have tested and expressed their thoughts on improving the project. The Museum has been live tested in Atlanta, Sarasota, Toulouse, Lyon, and Paris, within pubs, during university talks, and at friends' encounters, as shown in Figure 13.

Figure 13. Chosen user experiences in Atlanta and Toulouse

The easy way to fathom VR:

In addition to the interactive application, the 360 movies are pre-recorded scenarios. The viewer does not need to take any action and can enjoy the show (Wang et al., 2018). This could be a predefined path and simulated interactions. It has been reported as a relaxing experience, similar to and more powerful than a traditional 3D stereoscopic movie or a spherical theatre. It is well adapted to people who would be unfit for VR interactivity.

Viewing a 360-degree movie on a flat screen was recommended: it has not been designed for this purpose, and the experience is way too poor (Ozcinar & Smolic, 2018). Flat movies were produced and recorded within an environment where the camera and lighting were well-appropriated.

The users were categorized as follows:

Unfit users

The only problematic users are older adults who do not understand new technology or do not know English. Some users have difficulty dealing with it, have never played a computer game, or are more impressed by the headset than the application.

Moreover, some willing people seem sick with a headset; they told me it is a physical issue. The evolution of technology and opening more headsets may help them in the future.

Kids & teens

Kids usually like the environment and the color organization; they see “some logic” behind it. Those able to read and count were able to discover more mathematics behind the models and ask questions about the numbers and notes. Since children love to press any button and handle unusual tasks, it helps me debug the application. Reactions were noted, such as: “It is logical,” “But notes are numbers,” “Please change this, " and “Please include a game,” which helped in debugging the application.

Music learners

Depending on their musical knowledge, the interactive models give them a Wahoo effect when they realize that music can be displayed this way. They generally wish for more interactions and possibilities to analyze existing or live music.

Mathemusicians and researchers

They are usually not gifted for visualization and have their model in equations or, in the best case, on a drawing. One of our greatest pleasures is seeing colleagues experience what they have imagined on paper in' Reality.' When a researcher is placed inside his creation, he describes his feeling like an architect discovering the building he invented.

The navigation and customization are somehow too complex for a primary user; however, researchers appreciate the mathematical possibilities and the interactive part for improving the model.
The visualization helps verify their theory and gives new ideas for improvement.

CONCLUSION

Although the first results are pleasing, expertise in the development and execution of compelling gameplay and public promotion is still needed. Collaboration with other professionals strengthens the educational and interface features.

Watching pre-recorded movies is a stress-free and handy approach to capturing consumers' attention who may be uncomfortable with VR interaction and gaming.

For experienced mathematicians, the VR experience delivers the impression of being immersed inside the model since the display fills the whole seeing field, allowing us to notice details not visible on computer displays and concentrate on particular portions of the animation. Thus, the spherical look of Tonnetze, tori, and spheres, caused by the curvature of the representation, gives these items realism.

The immersive experience is also impressive to the general audience, particularly individuals who do not understand music or mathematics. It is generally the first time they experience the surroundings. Adults typically question the approach utilized; however, younger generations are more likely to relax and enjoy the performance.

REFERENCES

Albini, G., & Antonini, S. (2009). Hamiltonian cycles in the topological dual of the tonnetz. *Communications in Computer and Information Science,* 1–10. https://doi.org/10.1007/978-3-642-02394-1_1

Alonso, A., Zappi, V., & Serafin, S. (2022). Modeling of violin sound radiation as a dynamically consistent system for real-time synthesis. *Physics-Based Animation for Soundscapes,* 187–208.

Amiot, E. (2013). The Torii of Phases. *Mathematics and Computation in Music,* 1–18. https://doi.org/10.1007/978-3-642-39357-0_1

Andreatta, M., & Baroin, G. (2016). An introduction on formal and computational models in popular music analysis and generation. *Aesthetics and Neuroscience*, 257–269. https://doi.org/10.1007/978-3-319-46233-2_16

Andreatta, M., & Baroin, T. (2016). Structures mathématiques de la musique. *Mathématiques et sciences humaines*, (1), 51-78.

Bailey, N. J., Stevens, C. J., & Hughes, D. (2022). Symphonica: A virtual reality toolkit promoting co-creative musical interaction for people with multiple and complex needs. *Frontiers in Rehabilitation Sciences,* 3.

Baroin, G. (2011). The planet-4d model: An original hypersymmetric music space based on graph theory. *Mathematics and Computation in Music*, 326–329. https://doi.org/10.1007/978-3-642-21590-2_25

Baroin, G., & de Gérando, S. (2022). When virtual reality helps fathom Mathemusical hyperdimensional models. Mathematics and Computation in Music, 86–98. https://doi.org/10.1007/978-3-031-07015-0_8

Baroin, G. & Calvet, A. (2019). Visualizing Temperaments: Squaring the Circle?, *Conference on Mathematics and Computation in Music*, 333-337,

Baroin, G. & de Gérando, S. (2012). Sons et représentation visuelle en hyperespace : l'hypersphère des spectres, *Les Cahiers de l'Institut International pour l'Innovation, la Création Artistique et la Recherche*, (3rd ed.)

Baroin, G. (2020). Music Mathematic and 4D, keynote during Virtuality Experience, Buenos Aires / Paris 04 Dec (www.virtuality.io)

Bown, O., Eldridge, A., & McCormack, J. (2019). Understanding procedural audio in the context of interactive media. *Routledge companion to sound and music in videogames,* 181–202.

Briot, J.-P., Hadjeres, G., & Pachet, F.-D. (2020). Deep learning techniques for music generation. *Computational Synthesis and Creative Systems*. https://doi.org/10.1007/978-3-319-70163-9

Cheng, C. I., Wakefield, G. H., Roberts, J. C., & Fels, S. (2022). Perceptual quality of spatial audio in virtual reality applications: A comprehensive review. *ACM Transactions on Applied Perception,* 19(2), 1–23.

Chew, E. (2002). The spiral array: An algorithm for determining key boundaries. *Music and Artificial Intelligence,* 18–31. https://doi.org/10.1007/3-540-45722-4_4

Cohn, R. (2000). Weitzmann's regions, my cycles, and Douthett's dancing cubes. *Music Theory Spectrum*, *22*(1), 89–103. https://doi.org/10.2307/745854

Crans, S. (2019). Generalized Tonnetze. *Journal of Mathematics and Music*, 13(1), 1-22.

de Gérando, S. : music from « le Labyrinthe du Temps », Performances since 2020.

Douthett, J., & Steinbach, P. (1998). Parsimonious graphs: A study in parsimony, contextual transformations, and modes of limited transposition. *Journal of Music Theory*, *42*(2), 241. https://doi.org/10.2307/843877

Forte, A. (1977). *The structure of atonal music.* Yale University Press.

Hofmann, J. E., & Zemplén, G. Á. (2020). Newton, isaac: Opticks or a treatise of the reflections, Refractions, inflections & colours of light. *Kindlers Literatur Lexikon (KLL)*, 1–2. https://doi.org/10.1007/978-3-476-05728-0_15445-1

Hook, J. (2011). Review of dmitri tymoczko,*a geometry of music: Harmony and counterpoint in the extended common practice* (Oxford University Press, 2011). *Spcial Issue: (Per)Form in(g) Rock*, *17*(3). https://doi.org/10.30535/mto.17.3.10

Hughes, J. R., Smaili, R. H., Johnson, K., & Sacks, J. (2021). Orbifold path models for voice leading. *Journal of Mathematics and Music*, 15(1), 1–30.

Jedrzejewski, F. (2000). Ivan Wyschnegradsky et la musique microtonale. Musique,musicologie et arts de la scène. Université de Paris 1 Panthéon-Sorbonne

London, J. (2020). Musical rhythm: *A cognitive and computational perspective*. MIT Press.

Mannone, M. (2018). Introduction to gestural similarity in music. an application of category theory to the Orchestra. *Journal of Mathematics and Music*, *12*(2), 63–87. https://doi.org/10.1080/17459737.2018.1450902

Mannone, M., & Compagno, A. (2020). A network-theoretical approach to ensemble music composition. *Mathematics and Computation in Music,* 48(317/338), 102–114.

Mazzola, G. (2002). *The Topos of Music*. https://doi.org/10.1007/978-3-0348-8141-8

Nacke, L. E., Grimshaw, M. N., & Lindley, C. A. (2020). More than a feeling: measurement of sonic user experiences and psychophysiology in a first-person shooter game. *Interacting with Computers*, 22(4), 317–331.

Ozcinar, C., & Smolic, A. (2018). Visual attention in omnidirectional video for virtual reality applications. *2018 Tenth International Conference on Quality of Multimedia Experience (QoMEX)*. https://doi.org/10.1109/qomex.2018.8463418

Papadopoulos, A. (2022). Musical universals and the embodied schemata that ground them. *Music & Science*, 1–17.

Pressnitzer, D., Suied, C., & Shamma, S. A. (2020) Auditory scene analysis: the sweet spot for musical timbre. *NeuroImage*, 18(2), 1–9.

Risset, J.-C. (1986). Pitch and rhythm paradoxes: Comments on '"auditory paradox based on fractal waveform". *The Journal of the Acoustical Society of America*, *80*(3), 961–962. https://doi.org/10.1121/1.393919

Shepard, R. N. (1964). Circularity in judgments of relative pitch. *The Journal of the Acoustical Society of America*, *36*(12), 2346–2353. https://doi.org/10.1121/1.1919362

Sims, J. (n.d.). *The Square Root of Love, Letter to Pi*, facebook.com/johnsimsprojects.

Toussaint, G. (2022). *The geometry of musical rhythm* Chapman and Hall/CRC Press.

Tymoczko, D. (2021). *A geometry of music: Harmony and counterpoint in the extended common practice*. Oxford University Press.

Tymoczko, D. (2022). Visualizing harmony. *Mathematics and Computation in Music*, 216–227.

Villena-Taranilla, R., Tirado-Olivares, S., Cózar-Gutiérrez, R., & González-Calero, J. A. (2022). Effects of virtual reality on learning outcomes in K-6 education: A meta-analysis. *Educational Research Review*, *35*, 100434. https://doi.org/10.1016/j.edurev.2022.100434

Vuust, P., Brattico, E., Seppänen, M., Näätänen, R., & Tervaniemi, M. (2022). The sound of music: Differentiating music from speech. *Neuroscience & Biobehavioral Reviews,* 132, 1–11.

Wakefield, G., & Garg, D. (2021). Using virtual reality and interactivity for learning harmony. *Music Perception: An Interdisciplinary Journal*, 38(3), 232–246.

Wang, G., Gu, W., & Suh, A. (2018). The effects of 360-degree VR videos on audience engagement: Evidence from the New York Times. *HCI in Business, Government, and Organizations*, 217–235. https://doi.org/10.1007/978-3-319-91716-0_17

Yust, J. (2020). Generalized*tonnetze*and*zeitnetze*, and the topology of music concepts. *Journal of Mathematics and Music*, *14*(2), 170–203. https://doi.org/10.1080/17459737.2020.1725667

Zacharias, S., & Velichova, D. (2000). Projection from 4D to 3D. *Journal for Geometry and Graphics, 4*(1), 55-69.