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Gap Analysis of Ford-Fulkerson Algorithm and Edmonds-Karp Algorithm as Machine Learning Approach for Augmentation Path in the Maximum Flow Problem

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Abstract: The maximum flow problem is popular with many researchers as it plays a significant role in many areas. This problem is about obtaining the maximum flow where there is one source and sink in the network flow. In solving maximum flow problems, many algorithms are applied such as the most used are the Ford-Fulkerson Algorithm and the Edmonds-Karp Algorithm. The goal of this study is to identify the gaps of these two algorithms and sufficiently provide an analysis and comparison of their time complexity, simplicity, and effectiveness to solve the augmentation path in the maximum flow problem. The Ford-Fulkerson and Edmonds-Karp algorithms performed well and sufficiently to solve the augmentation path in the maximum flow problem. However, both algorithms consist of different gaps like having slow time complexity and difficulty in implementation and execution.

***Keywords**:* Maximum Flow Problem, Augmentation Path, Ford Fulkerson Algorithm, Network Flow, Edmonds-Karp Algorithm

INTRODUCTION

Network flow is an optimization of the wide variety of management and engineering problems that affect how objects move through a network. There are many things that network flow has been applied to, such as communication networks, transportation systems, and some queuing systems. This network flow causes some problems in both running time and its space complexity. The maximum flow in a network is one of the problems in solving the augmentation path. Therefore, it means that solving the maximum flow problem is very important in many fields or applications, that's why many researchers studied how this problem will be optimized.

The directed graph of an optimized stationary network flow is the goal of the maximum flow problem. The optimizing transit into a constant maximum rate is the load flow from the single source of a vertex into a single sink vertex via a network that has a maximum carrying capacity on each edge. The problem has received extensive attention in the graph of the theory where it is focused on real-world applications, such as scheduling, prediction, and the segmentation of an image (Krauss, McCollum, Pendery, Litwin, & Michaels, 2020).

Many algorithms have been used to solve the augmentation path of the maximum flow problem, such as Dinic's Algorithm, Incremental Algorithm, and the Max-cost min-cut algorithm. The researchers chose Ford-Fulkerson Algorithm and Edmonds-Karp Algorithm to evaluate its gaps when applied to solve the maximum flow problem (Kyi, & Naing, 2018). The Ford-Fulkerson Algorithm was created by Ford, L. and Fulkerson, D. in the late 1950s. The algorithm begins with the start of the flow and the sequence of the recursive constructs in the increasing number of the flow and will ends with the maximum value. The idea of this algorithm is that the path of the flow that we send from the source sets a graph that has capacity at the edges of the path, also known as augmenting the path (Jiang, Hu & Gao, 2013). After that, they found another path until the end of the flow.

The researchers also used the Edmonds-Karp Algorithm, which help to lessen the issues in the maximum flow problem. The Edmonds-Karp algorithm was also created by Ford-Fulkerson, but it was proposed in early 1972 by Edmond, Jack, and Karp, Richard. It is a method in a network flow for the calculation of the time in order to determine the maximum flow problem. It is also an algorithm that has the capacity to find the augmentation paths from the source to the terminal. The Edmonds-Karp algorithm process will be terminated once the possible paths have the nonzero capability. In terms of large paths, it is very slow to run, but as of now, it has a version that optimizes this problem (Chaibou, Tessa, Sié, & Faso, 2019). This paper aims to analyze the gaps of the Ford-Fulkerson Algorithm and the Edmonds-Karp Algorithm in solving the maximum flow problem and provide a comparative analysis between the two algorithms.

LITERATURE SURVEY

The networking and communication areas have significant applications in the maximum flow problem. In dealing with the max flow algorithms, the simple path is applied where these simple paths have vertices that have source and sink that are used in calculating the max flow (Wei, Liu, & Zhang, 2018). According to Shi, Sheng, and Ralescu, (2017), the optimization of a network is a critical topic in the maximum flow problem, with engineering and management applications spanning a wide range of disciplines. The problem aims to discover the highest value of flow in a network from the source to the sink (Khanafseh, Surakhi, Sharieh, & Sleit, 2017). The max flow algorithm has two categories such as the augmentation of the path and the pre-flow push. The augmenting path is where the path will be found from the start to the end of the flow and then will send the flow into the paths found (Pang et al., 2018; Peng, Lin, Huang, Zou, & Yang, 2017). These paths are from the networks that have simple paths wherein the original network the damaged network was included (Wang et al., 2018). While the pre-flow push is a flood graph where there is an incoming and outgoing flow of nodes without the constraint of the mass balance (Pang et al., 2018).

Many algorithms have been proposed to solve the max flow problems. Most of the max flow algorithms that have been used are the Ford-Fulkerson. In this algorithm, it used augmenting paths in the residual network. It also accumulated the flow in an augmented until there was no path in the network of the residual. In addition, the max flow algorithm also has Dinic, the shortest path combination algorithm, and the Edmonds-Karp algorithm. These algorithms have the same purpose of maximizing the flow of the augmented path, but they also have different features that were improved to solve the max flow problems (Wang et al., 2018).

The most well-known methods for determining maximum flow issues include the Ford-Fulkerson algorithm and the Edmonds-Karp algorithm. Even though they are both algorithms used for maximizing flow, their particular characteristics differ. Ford-Fulkerson finds augmenting routes across a residual graph using depth-first searches, whereas Edmond-Karp utilizes breadth-first searches to obtain polynomial computation time (Kyi, & Naing, 2018).

In a study conducted by Surakhi, Qatawneh, and Al Ofeishat (2017), the Ford Fulkerson algorithm is one of the most used approaches for solving the maximum flow problem; however, it is somewhat complex, and the time complexity is very high. Having said that, for the researchers to solve the maximum flow, the parallel genetic algorithm was applied wherein there is a simultaneously directed graph in the cycle which determines the objective function value on each parallel step of an augmenting path is applied from the source to the sink. The implementation was carried out on the IMAN1 supercomputer utilizing the open MPI library. The results show that it sped up the running time or the time complexity with up to 50% parallel efficiency.

According to the study by Bai et al. (2018), the researchers applied the Ford-Fulkerson algorithm to test the Interconnect Resource. It is one of the important parts of the Field Programmable Gate Arrays or FPGA. The use of

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the Ford-Fulkerson algorithm reduces the expansion of the scale in FPGA. The study helps to improve the use of the Ford-Fulkerson algorithm which the FGPA is divided into three graphs consisting of horizontal, vertical, and oblique graphs that help to reduce the complexity of the large scales of the FPGA.

In Kyi and Naing (2018), the maximum possible flow of Pyigyitagon Township's water distribution pipeline network was determined using the Ford-Fulkerson approach from flow graph theory. Based on observation, Ford-Fulkerson's calculation time is much higher than other algorithms mentioned. However, the termination was guaranteed in this algorithm when it has the simplest to implement, and if the edge capacities are non-negative real numbers. Having said that, it was strongly recommended to solve the maximum flow problem.

Jain and Garg (2012) stated that the Edmonds-Karp technique is a well-known algorithm for solving the max-flow problem. In their research, the goal was to identify the shortest augmenting pathways in the existing residual network until no augmenting paths can be identified. Implementing map reduction based on the Edmonds-Karp algorithm implementation worked well when the capacities are integral, and it has considerably less time complexity than the Ford-Fulkerson technique. The Edmonds-Karp algorithm was able to deliver a more effective and faster complexity run time in the average case and best case.

In the study of Rajalakshmi and Vaidyanathan (2019), the researchers used the Edmonds-Karp Algorithm in a research experiment. The study was about a Traffic Management System that aims to reduce traffic congestion in the streets. This algorithm was used to solve the max-flow problems in planning, artificial vision, operational research, and optimization of the network (Chaibou, Tessa, Sié, & Faso, 2019). It also provides a solution in a flow network to find the maximum amount of flow. The process of this algorithm is repeated until no path in the source to sink exists.

PROBLEM DOMAIN

Based on the listed research works, the researchers were able to gather all the problems encountered by the previous researchers regarding the two algorithms used. The following are:

1. The capacity of augmentation paths should be modified and have criteria in the selection of the limits of the max flow problem (Singh, Joshi, & Kohli, 2015).
2. The Edmonds-Karp Algorithm needs some improvements such as the automated network should be extended into subnetworks that each have one source and sink (Rajalakshmi & Vaidyanathan, 2019).
3. Improve the bounds, estimated number of augmentations, and a strong polynomial-time algorithm in matrices of the Edmonds-Karp Algorithm (De Loera, Hemmecke, & Lee, 2015).
4. The Ford-Fulkerson Algorithm takes a long time or high time complexity to solve/calculate for the nearest correct path distance.
5. The Ford-Fulkerson Algorithm is greedy (Faigle, Kern, & Peis, 2018) making it not suitable for all the areas where the max-flow problem is involved.
6. The Ford-Fulkerson Algorithm also has a problem with execution, it tends not to terminate when the augmenting path was chosen poorly (Biswas, Sundar, & Paul, 2007).

COMPARISON AND ANALYSIS

This section discusses the detailed definition of the algorithms used and the comparison and gaps gathered through several types of research.

A. Ford-Fulkerson Algorithm

Ford-Fulkerson Algorithm is used in network flow to compute the maximum flow (Haque, & Isla, 2020). The algorithm's concept is straightforward. We send flow via an augmenting path, in which there is an accessible capacity in of its edges and the path will flow from source to sink in a graph and then we identify another path, and so on.

```
Pseudocode: Ford-Fulkerson Algorithm
Start and initialize the flow "f" to zero(0)
Check if there is an augmenting path from source to sink.
Loop until the condition above is true
        Continuously add this path-flow to the flow.
Return.
```

Fig. 1. Pseudocode for Ford-Fulkerson Algorithm

The Ford-Fulkerson algorithm consists of two steps. In the first step, find the flow in the labeling procedure of an augmenting path. Lastly, adjust the flow correspondingly until there is no augmenting path in the flow of the network.

B. Edmonds-Karp Algorithm

The Edmonds-Karp Algorithm is a computing method in the flow network of the maximum flow and the approach is more optimized. It has a well-defined search order of augmenting paths. The augmenting path of this algorithm is along the shortest path which leads to the polynomial-time algorithm (Akter, Uddin, & Shami, 2021).

```
Pseudocode: Edmonds-Karp Algorithm

Check if a path from s to t in the residual graph exists.
While the condition above is true:
    Find shortest amount of flow in the augmenting
    path P between s and t in residual graph
    Find the maximum amount of flow fmax, push along P
Update residual graph using path P and maximum flow fmax
```

Fig. 2. Pseudocode for Edmonds-Karp Algorithm

The operations of the Edmonds-Karp algorithm are repeated until it will end the augmenting path. The sum of the distinct paths s and t is equal to the maximum flow. First, it will initialize the flow of the value on each edge. The flow in the capacity of the residual will be increased until all paths will be saturated. This algorithm will solve the max flow problem by starting the process from the value of zero as long as the paths of the flow will be increasing (Chaibou, Tessa, Sié, & Faso, 2019).

The Edmonds-Karp Algorithm has many modified versions that are being studied. This approach is faster since it requires fewer iterations and augmentations to obtain the maximum flow. This implies that the optimization in the runtime of the max flow was better than other algorithms.

Table 1.

Algorithm Comparisons

Algorithm	Time Complexity	Simplicity	Effectiveness
Ford-Fulkerson	Slow	Simple to Implement	Effective
Edmonds-Karp	Fast	Complex to Implement	Very Effective

Edmonds-Karp Algorithm was much faster compared to Ford-Fulkerson Algorithm when solving the augmentation path in Maximum Flow Problem, this is because it is a derived and an improved version of the Ford-Fulkerson. When it comes to simplicity, according to the studies, Ford-Fulkerson is more straightforward to implement than the Edmonds-Karp, making it the first choice for some other researchers. For effectiveness, both algorithms are effective in solving the problem, however, in some studies, it has been proven that Ford-Fulkerson has a terminating problem when the augmented path is chosen poorly.

CONCLUSION AND FUTURE WORK

Maximum Flow Problem in network flow is a critical topic that many researchers are interested in solving as it can help solve many different areas, such as, but not limited to, traffic management, forestry, water distribution, and more. In this paper, two of the most commonly known algorithms are discussed: the Ford-Fulkerson Algorithm and the Edmonds-Karp Algorithm. The gap analysis for the two algorithms concludes that both effectively solve the augmentation path in the maximum flow problem. However, only Edmonds-Karp performed efficiently and quickly when it came to execution. Even though Ford-Fulkerson's time complexity is slow compared to other algorithms, other researchers still prefer it as it is easier to implement than the modified version, Edmonds-Karp.

For future work, researchers can apply other max-flow algorithms such as Incremental Algorithm and Max-Cost Min-Cut Algorithm to solve the max flow problems. Make sure that the algorithm used is appropriate for the study. In addition, there are some modified versions of the Ford-Fulkerson and Edmonds-Karp Algorithms, which can help more to optimize the max flow problems.

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